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CONTACT PROBLEM OF ELASTICITY THEORY FOR A WEDGE*

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An exact closed solution of the plane static contact problem of elasticity theory on the impression of a rigid stamp of arbitrary form on an elastic wedge is constructed by the Wiener—Hopf method. Particular cases of plane and parabolic stamps are examined in more detail. For a wedge angle greater than π the problem belongs to the class N and the external field must be taken into account for a correct solution. In this case, the nontrivial solution of the corresponding homogeneous problem is constructed and investigated.

1. Formulation of the problem. Let us consider an infinite elastic wedge $0 < r < \infty$, $0 < \theta < \alpha$ $(0 < \alpha < 2\pi)$ at whose edge $\theta = 0$ a rigid stamp is impressed (Fig.1). There is no friction between the stamp and the wedge surfaces. It is assumed that the section $\theta = 0$, $0 < r < l$ of the elastic body boundary is in contact with the stamp. The wedge surface is stress-free outside the line of contact.

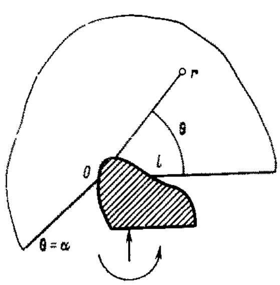

Fig.1

In the case when the wedge opening angle α is greater than π, the problem under consideration belongs to the class N. Let us recall /1,2/ that the Saint-Venant principle is not satisfied in the class N and a nontrivial damped solution of homogeneous problems exists which is much greater than the inhomogeneous solution (external field) in absolute value, at great distances. The Saint-Venant principle is valid in the well-investigated class S, and nontrivial damped solutions of the homogeneous problems does not exist. Hence, for $\alpha > \pi$ the solution of the problem under consideration behaves at infinity as the greatest solution of the homogeneous problem satisfying the stress damping condition asymptotically for a wedge $0 < r < \infty$, $0 < \theta < \alpha$ with stress-free edges. This latter is determined to the accuracy of two arbitrary real constants C_I and C_{II}. These constants are considered given by the condition of the problem. They characterize the external field intensity which exerts substantial influence on the state of stress in an elastic body with an infinitely remote point in this case. In the case under consideration the stresses damp out at infinity more slowly than $O(1/r)$.

If $\alpha \leqslant \pi$ then the problem belongs to the class S, the external field exerts no substantial influence on the state of stress in the wedge, and the stresses damp out at infinity as $O(1/r)$.

The boundary conditions of the problem under consideration can be written as follows:

$$\theta = \alpha, \quad \sigma_\theta = \tau_{r\theta} = 0; \quad \theta = 0, \quad \tau_{r\theta} = 0 \tag{1.1}$$

$$\theta = 0, \quad r > l, \quad \sigma_\theta = 0; \quad \theta = 0, \quad r < l, \quad u_\theta = f(r) \tag{1.2}$$

$$\theta = 0, \quad r \to \infty, \quad \partial u_\theta / \partial r = O(1/r) \quad (0 < \alpha \leqslant \pi) \tag{1.3}$$

$$\frac{\partial u_\theta}{\partial r} = -\frac{2(1-\nu^2)}{E} C_I (2\pi r)^{\lambda_1 - 1} \sin\frac{(\lambda_1 - 1)\alpha}{2} + O\left(\frac{1}{r}\right) \quad (\pi < \alpha \leqslant \alpha_*)$$

$$\frac{\partial u_\theta}{\partial r} = -\frac{2(1-\nu^2)}{E} C_I (2\pi r)^{\lambda_1 - 1} \sin\frac{(\lambda_1 - 1)\alpha}{2} - \frac{1+\nu}{2E} C_{II} (2\pi r)^{\lambda_2 - 1} \times$$
$$\frac{(2-4\nu)\cos(\lambda_2 - 1)\alpha/2 + (\lambda_2 + 1)\sin\alpha}{\sin(\lambda_2 + 1)\alpha/2} + O\left(\frac{1}{r}\right) \quad (\alpha_* < \alpha < 2\pi)$$

$$\theta = 0, \quad r \to l - 0, \quad \sigma_\theta \sim \frac{K}{\sqrt{2\pi(l - r)}} \tag{1.4}$$

$$\int_0^l \sigma_\theta(r, 0)\, dr = Y, \quad \int_0^l \sigma_\theta(r, 0)\, r\, dr = M \tag{1.5}$$

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Here σ_θ, $\tau_{r\theta}$, σ_r are the stresses, u_θ, u_r are displacements, $\lambda_j(\alpha) \in (1/2, 1)$ $(j = 1, 2)$ is the single root of the equation $\sin p\alpha - (-1)^j p \sin\alpha = 0$ in the strip $0 < \operatorname{Re} p < 1$ of the complex plane p; $C_{\rm I}(\alpha)$ $(\pi \leqslant \alpha < 2\pi)$ and $C_{\rm II}(\alpha)$ $(\alpha_* \leqslant \alpha < 2\pi)$ are given continuous functions α for which $C_{\rm I}(\pi) = 0$, $C_{\rm II}(\alpha_*) = 0$; α_* is the single root of the equation $\alpha\cos\alpha - \sin\alpha = 0$ in the interval $\pi < \alpha < 2\pi$; $f(r)$ is a given function, K is a factor to be determined, $(0, Y)$ and M are the principal vector and principal moment of the force in the section $\theta = 0, 0 < r < l$ (the quantity Y is given, but M should be determined during solution of the problem), E is Young's modulus, and ν is the Poisson's ratio.

If the length l of the contact line is not known, it is determined from the condition that the coefficient K is zero (here the stresses will be constrained at the point $\theta = 0, r = l$). Without limiting the generality, the length of the contact line can be considered one.

The problem formulated is a particular case of a contact problem for a wedge with contact section $\theta = 0$, $d < r < d + l$ for $d = 0$. The solution of this problem is contructed in /3/ in the absence of an external field. Let us note that the first fundamental problem of elasticity theory for a wedge with opening angle greater than π was first considered in /4/.

We present certain information about the roots of the equation $\sin 2p\alpha + p\sin 2\alpha = 0$ $(0 < \alpha < 2\pi)$ in the strip $0 < \operatorname{Re} p < 1$.

For $0 < \alpha \leqslant \pi/2$ this equation has no roots in the strip mentioned. For $\pi/2 < \alpha \leqslant \pi$ a single root $\mu_1 \in (1/2, 1)$ exists, for $\pi < \alpha \leqslant 3\pi/2$ two roots $\mu_1 \in (0, 1/2)$ and $\mu_2 \in (1/2, 1)$ while for $3\pi/2 < \alpha < 2\pi$ three roots, $\mu_1 \in (0, 1/2)$, $\mu_2 \in (1/2, 1)$ and $\mu_3 \in (1/2, 1)$, where $\mu_2 < \mu_3$.

2. Solution of the inhomogeneous problem without taking account of the external field. Let there be no external field ($C_{\rm I} = 0$, $C_{\rm II} = 0$).

Applying the Mellin integral transformation

$$m^*(p) = \int_0^\infty m(r) r^p\, dr$$

to the equilibrium equations, the strain compatibility condition, and the "through" conditions (1.1), we obtain /5/:

$$\sigma_\theta^*(p, \theta) = A_1 \sin(p+1)\theta + A_2 \sin(p-1)\theta + A_3\cos(p+1)\theta + A_4\cos(p-1)\theta \tag{2.1}$$

$$\tau_{r\theta}^* = (p-1)^{-1}\frac{d\sigma_\theta^*}{d\theta}, \quad p\sigma_r^* = (p-1)^{-1}\frac{d^2\sigma_\theta^*}{d\theta^2} - \sigma_\theta^*$$

$$\begin{aligned} &A_1\sin(p+1)\alpha + A_2\sin(p-1)\alpha + A_3\cos(p+1)\alpha + A_4\cos(p-1)\alpha = 0 \\ &A_1(p+1)\cos(p+1)\alpha + A_2(p-1)\cos(p-1)\alpha - A_3(p+1)\sin(p+1)\alpha - A_4(p-1)\sin(p-1)\alpha = 0 \\ &A_1(p+1) + A_2(p-1) = 0 \end{aligned} \tag{2.2}$$

From (2.2) we find

$$A_2 = -\frac{p+1}{p-1}A_1, \quad A_3 = -\frac{2(p\sin^2\alpha + \sin^2 p\alpha)}{\sin 2p\alpha + p\sin 2\alpha}A_1 \tag{2.3}$$
$$A_4 = -\frac{2(p+1)(p\sin^2\alpha - \sin^2 p\alpha)}{(p-1)(\sin 2p\alpha + p\sin 2\alpha)}A_1$$

In conformity with (2.1) and (2.3)

$$\sigma_\theta^*(p, 0) = -\frac{4(p^2\sin^2\alpha - \sin^2 p\alpha)}{(p-1)(\sin 2p\alpha + p\sin 2\alpha)}A_1 \tag{2.4}$$

Using Hooke's law, taking account of (2.1) and (2.3), we obtain

$$\int_0^\infty \left.\frac{\partial u_\theta}{\partial r}\right|_{\theta=0} r^p\, dr = -\frac{4(1-\nu^2)}{E}(p-1)^{-1}A_1 \tag{2.5}$$

Eliminating A_1 in the relationships (2.4) and (2.5), and taking into account the "dual" conditions (1.2), we arrive at a Wiener–Hopf functional equation:

$$\Phi^-(p) = -\mathrm{tg}\, p\pi G(p)\,[\Phi^+(p) + g(p)] \quad (-\mu < \mathrm{Re}\, p < 0) \tag{2.6}$$

$$G(p) = \frac{2(p^2\sin^2\alpha - \sin^2 p\alpha)}{-\mathrm{tg}\, p\pi\,(\sin 2p\alpha + p\sin 2\alpha)}$$

$$\Phi^-(p) = \int_0^1 \sigma_\theta(r, 0)\, r^p\, dr, \quad \Phi^+(p) = \frac{E}{2(1-\nu^2)}\int_1^\infty \frac{\partial u_\theta}{\partial r}\Big|_{\theta=0} r^p\, dr$$

$$g(p) = \frac{E}{2(1-\nu^2)}\int_0^1 f'(r)\, r^p\, dr, \quad \mu = \begin{cases} 1, & 0 < \alpha \leqslant \pi/2 \\ \mu_1, & \pi/2 < \alpha < 2\pi \end{cases}$$

($f'(r)$ is the derivative of the function $f(r)$ with respect to r). The method of isolating $\mathrm{tg}\, p\pi$ in the coefficient of the Wiener–Hopf equation permits solving the problem of factorizing the function with an essential singularity at infinity. It was used first in /6/, and then in many papers related to the utilization of the Mellin transform and the Wiener–Hopf method.

The following factorizations are valid for the functions $G(p)$ and $\mathrm{tg}\, p\pi$ /7/:

$$G(p) = G^+(p)/G^-(p) \quad (\mathrm{Re}\, p = 0) \tag{2.7}$$

$$\exp\left[\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \frac{\ln G(t)}{t-p}\, dt\right] = \begin{cases} G^+(p), & \mathrm{Re}\, p < 0 \\ G^-(p), & \mathrm{Re}\, p > 0 \end{cases}$$

$$\mathrm{tg}\, p\pi = \frac{p}{K^+(p)K^-(p)}, \quad K^\pm(p) = \frac{\Gamma(1 \mp p)}{\Gamma(1/2 \mp p)}$$

($\Gamma(z)$ is the Euler gamma-function).

Taking account of (2.7) and using the representation

$$G^+(p)\, g(p)\, [K^+(p)]^{-1} = g^+(p) - g^-(p) \quad (\mathrm{Re}\, p = 0) \tag{2.8}$$

$$\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \frac{G^+(t)\, g(t)}{K^+(t)}\frac{dt}{t-p} = \begin{cases} g^+(p), & \mathrm{Re}\, p < 0 \\ g^-(p), & \mathrm{Re}\, p > 0 \end{cases}$$

we obtain from (2.6)

$$\Phi^-(p)\, K^-(p)\, G^-(p) - p g^-(p) = -p\Phi^+(p)\, G^+(p)\, [K^+(p)]^{-1} - p g^+(p) \quad (\mathrm{Re}\, p = 0) \tag{2.9}$$

The function in the left side of (2.9) is analytic in the half-plane $\mathrm{Re}\, p > 0$ and the function in its right side is analytic in the half-plane $\mathrm{Re}\, p < 0$. On the basis of the principle of analytic continuation they equal the same function that is analytic in the whole plane p. It follows from (1.4), (2.7) and (2.8) that the function in the left side of (2.9) tends to the constant

$$\frac{K}{\sqrt{2}} - \delta, \quad \delta = -\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \frac{G^+(p)\, g(p)}{K^+(p)}\, dp$$

in the half-plane $\mathrm{Re}\, p > 0$ as $p \to \infty$.

Therefore, by the Liouville theorem a single analytic function is identically equal to this constant in the whole plane p. In particular

$$\Phi^-(0)\, K^-(0)\, G^-(0) = \frac{K}{\sqrt{2}} - \delta$$

Taking into account (1.5), we obtain from the last equation

$$K = (\delta + cY)\sqrt{2}, \quad c = \left[\frac{2\alpha + \sin 2\alpha}{2(\alpha^2 - \sin^2\alpha)}\right]^{1/2} \tag{2.10}$$

The solution of the functional equation (2.6) has the form

$$\Phi^-(p) = \frac{p g^-(p) + cY}{K^-(p)\, G^-(p)} \quad (\mathrm{Re}\, p > 0)$$

$$\Phi^+(p) = -\frac{p g^+(p) + cY}{p G^+(p)} K^+(p) \quad (\mathrm{Re}\, p < 0)$$

3. The homogeneous problem.

Let us assume that a wedge with opening angle greater than π is deformed only because of the external field; the normal displacement and the principal force vector are zero on the section $\theta = 0$, $0 < r < 1$.

The solution of this homogeneous problem equals the sum of the solutions of the following two problems. The first is a particular case of the boundary value problem considered in

Sect.2, for $Y = 0$ and

$$f'(r) = \frac{2(1-\nu^2)}{E} C_{\rm I}(2\pi r)^{\lambda_1-1}\sin\frac{(\lambda_1-1)\alpha}{2} + \frac{1+\nu}{2E} C_{\rm II}(2\pi r)^{\lambda_2-1}\frac{(2-4\nu)\cos(\lambda_2-1)\alpha/2+(\lambda_2+1)\sin\alpha}{\sin(\lambda_2+1)\alpha/2} \quad (3.1)$$

(for $\pi < \alpha \leqslant \alpha_*$ there is no component corresponding to $C_{\rm II}$). The second problem is the homogeneous problem for a wedge with stress-free edges (the solution of this last is kept in mind, which is realized as the asymptotic of the solution of the initial problem at infinity (see (1.3))).

4. **Analysis of the solutions.** The complete solution of the initial problem is the sum of the particular solution of the inhomogeneous problem constructed in Sect.2 and the homogeneous solution of Sect.3.

Let us determine the length l of the contact line in the initial problem for a smooth stamp of arbitrary shape from the condition $K = 0$. According to (2.10) and (3.1), by going over to dimensional variables we arrive at the following equation

$$\delta_1 l^{1/2} + cYl^{-1/2} - \frac{G^+(-\lambda_1)}{K^+(-\lambda_1)} Z_1 C_{\rm I} l^{\lambda_1-1/2} - \frac{G^+(-\lambda_2)}{K^+(-\lambda_2)} Z_2 C_{\rm II} l^{\lambda_2-1/2} = 0$$

(for $\pi < \alpha \leqslant \alpha_*$ there is no component corresponding to $C_{\rm II}$ while for $0 < \alpha \leqslant \pi$ none corresponding to $C_{\rm I}$ and $C_{\rm II}$).

Here

$$\delta_1 = -\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{G^+(p)g_1(p)}{K^+(p)}dp, \quad g_1(p) = \frac{E}{2(1-\nu^2)}\int_0^1 f'(\rho l)\rho^p d\rho$$

$$Z_1 = (2\pi)^{\lambda_1-1}\sin\frac{(\lambda_1-1)\alpha}{2}$$

$$Z_2 = (2\pi)^{\lambda_2-1}\frac{(2-4\nu)\cos(\lambda_2-1)\alpha/2+(\lambda_2+1)\sin\alpha}{4(1-\nu)\sin(\lambda_2+1)\alpha/2}$$

We present the asymptotic formula for the contact stress σ_θ $(\theta = 0, 0 < r < l)$ in the neighborhood of the angular point (for $r \to 0$):

$$\sigma_\theta = \sum_{k=1}^{n} h_k\left\{g_1(-\mu_k) + \frac{K^+(-\mu_k)}{G^+(-\mu_k)}\left[\frac{cYl^{-1}-\mu_k g_k^+}{\mu_k} + \frac{Z_1}{\lambda_1-\mu_k}\frac{G^+(-\lambda_1)}{K^+(-\lambda_1)}C_{\rm I}l^{\lambda_1-1} + \frac{Z_2}{\lambda_2-\mu_k}\frac{G^+(-\lambda_2)}{K^+(-\lambda_2)}C_{\rm II}l^{\lambda_2-1}\right]\right\}\left(\frac{r}{l}\right)^{\mu_k-1} + R$$

(for $\pi < \alpha \leqslant \alpha_*$ there is no component corresponding to $C_{\rm II}$, while for $0 < \alpha \leqslant \pi$ none corresponding to $C_{\rm I}$ and $C_{\rm II}$).

Here

$$h_k = \frac{2(\mu_k^2\sin^2\alpha - \sin^2\mu_k\alpha)}{2\alpha\cos 2\mu_k\alpha + \sin 2\alpha}$$

$$g_k^+ = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{G^+(p)g_1(p)}{K^+(p)}\frac{dp}{p+\mu_k}, \quad n = \begin{cases}1, & 0<\alpha\leqslant\pi\\ 2, & \pi<\alpha\leqslant 3\pi/2\\ 3, & 3\pi/2<\alpha<2\pi\end{cases}$$

$$R = \begin{cases} o(r^{\mu_1-1}), & 0<\alpha\leqslant\pi/2\\ O(r^\gamma), & \pi/2<\alpha<2\pi\end{cases}$$

$(\gamma \geqslant 0, \gamma = 0$ only for $\alpha = 3\pi/2)$.

Let us determine the domain of existence of the contact problem solution constructed by considering, as is usual, a negative pressure to be impossible at the contact line. For $f'(r) \leqslant 0$ when the angular point of the body lies on the contact line starting with the very first moment of contact between the stamp and the body, the contact pressure will everywhere be positive. If the function $f'(r)$ changes sign once from plus to minus, the solution will exist only starting with certain values of the parameters of the problem governed by the following conditions:

$$g_1(-\mu_1) + \frac{K^+(-\mu_1)}{G^+(-\mu_1)}\left[\frac{cYl^{-1}-\mu_1 g_1^+}{\mu_1} + \frac{Z_1}{\lambda_1-\mu_1}\frac{G^+(-\lambda_1)}{K^+(-\lambda_1)}C_{\rm I}l^{\lambda_1-1} + \frac{Z_2}{\lambda_2-\mu_1}\frac{G^+(-\lambda_2)}{K^+(-\lambda_2)}C_{\rm II}l^{\lambda_2-1}\right] = 0 \quad (4.1)$$

(for $\pi < \alpha \leqslant \alpha_*$ there is no component corresponding to $C_{\rm II}$, and for $0 < \alpha \leqslant \pi$ none corresponding to $C_{\rm I}$ and $C_{\rm II}$).

This equation determines the time when the contact pressure changes sign at the angular point; it corresponds to the principal term vanishing in the expansion of the stress at the angular point. Condition (4.1) is an extension of the known condition of N.I. Muskhelishvili that requires boundedness of the stress at the end of the contact line (this latter was used above in defining l).

By using (1.5) we obtain the following expression for the moment M:

$$M = \frac{\sqrt{\pi}}{2G^-(1)}\left[g^- l^2 + cYl - \frac{Z_1}{1+\lambda_1}\frac{G^+(-\lambda_1)}{K^+(-\lambda_1)}C_{\rm I} l^{\lambda_1+1} - \frac{Z_2}{1+\lambda_2}\frac{G^+(-\lambda_2)}{K^+(-\lambda_2)}C_{\rm II} l^{\lambda_2+1}\right]$$

$$g^- = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{G^+(p)\,g_1(p)}{K^+(p)}\,\frac{dp}{p-1}$$

(for $\pi < \alpha \leqslant \alpha_*$ the component corresponding to $C_{\rm II}$, and for $0 < \alpha \leqslant \pi$ the components corresponding to $C_{\rm I}$ and $C_{\rm II}$ are missing).

Let us note certain particular cases of the problem under consideration in the case of no external field.

5. Stamp with rectilinear horizontal base. In this case $f'(r) = 0$, and the length l of the contact line is known.

In this case the contact problem is analogous to the corresponding problem for a wedge $-\alpha < \theta < \alpha$ with a slit for $\theta = 0,\ r > l$. Hence, for $0 < \alpha \leqslant \pi$ the results obtained here agree with the solution of the problem mentioned obtained in /8/. For $\pi < \alpha < 2\pi$ they yield the solution of the problem for the corresponding two-layer elastic plate with a slit for $\theta = 0,\ r > l$ if friction between the overlying plates is neglected in the domain $2\pi - \alpha < \theta < \alpha$. In these problems the evaluation of the factor

$$K = \left(\frac{2\alpha + \sin 2\alpha}{\alpha^2 - \sin^2\alpha}\right)^{1/2} Y l^{-1/2}$$

is of main interest for fracture mechanics.

Let us present the formula for the contact stress under the stamp ($\theta = 0,\ 0 < r < l$):

$$\sigma_\theta = \frac{cYl^{-1}}{\pi i}\int_{-i\infty}^{i\infty}\frac{S(p)\,K^+(p)}{pG^+(p)}\left(\frac{r}{l}\right)^{-p-1}dp$$

$$S(p) = \frac{\sin^2 p\alpha - p^2\sin^2\alpha}{\sin 2p\alpha + p\sin 2\alpha}$$

6. Stamp with rectilinear sloping base. Let $f'(r) = a$ (a is a dimensionless contact). In this case the equation to determine the length of the contact line has the form

$$cYl^{-1/2} - \frac{aE}{4(1-\nu^2)}G^+(-1)(\pi l)^{1/2} = 0$$

We hence find

$$l = \frac{4(1-\nu^2)}{E}c\pi^{-1/2}[G^+(-1)]^{-1}Ya^{-1}$$

The contact stress under the stamp is expressed by the formula

$$\sigma_\theta = \frac{1}{\pi i}\int_{-i\infty}^{i\infty} S(p)\left[cYl^{-1}p^{-1} - \frac{aE}{4(1-\nu^2)}\pi^{1/2}G^+(-1)(p+1)^{-1}\right]\times\frac{K^+(p)}{G^+(p)}\left(\frac{r}{l}\right)^{-p-1}dp$$

7. Parabolic stamp. Let $f'(r) = br/l$ (b is a dimensionless constant). The equation to determine the length of the contact line is written thus

$$cYl^{-1/2} - \frac{3bE}{16(1-\nu^2)}G^+(-2)(\pi l)^{1/2} = 0$$

We hence obtain

$$l = \frac{16(1-\nu^2)}{3E}c\pi^{-1/2}[G^+(-2)]^{-1}Yb^{-1}$$

The formula for the contact stress has the form

$$\sigma_\theta = \frac{1}{\pi i}\int_{-i\infty}^{i\infty} S(p)\left[cYl^{-1}p^{-1} - \frac{3bE}{16(1-\nu^2)}\pi^{1/2}G^+(-2)(p+2)^{-1}\right]\times\frac{K^+(p)}{G^+(p)}\left(\frac{r}{l}\right)^{-p-1}dp$$

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